Analytic in the unit ball vector-functions having bounded $\mathbf{L}$-index in joint variables

Vita Baksa, Andriy Bandura and Oleh Skaskiv

We will use denotations from [?]. Particularly, by $\mathbb{D}^2((z_0,\omega_0),R)$ we denote a polydics $\{(z,\omega)\in\mathbb{C}^2: |z-z_0|<r_1, |\omega-\omega_0|<r_2\}$, $\mathbb{B}^2((z_0,\omega_0),r)=\{(z,\omega)\in\mathbb{C}^2: \sqrt{|z-z_0|^2+|\omega-\omega_0|^2}<r\}$, $\mathbb{B}^2=\mathbb{B}^2((0,0),1)$.

Let $\mathbf{L}(z,\omega)=(l_1(z,\omega),l_2(z,\omega))$, where $l_j(z,\omega):\mathbb{B}^2\to\mathbb{R}_+$ is a positive continuous function such that

$$\forall (z,\omega)\in\mathbb{B}^2: \quad l_j(z,\omega)>\frac{\beta}{1-\sqrt{|z|^2+|\omega|^2}},\ j\in\{1,2\}, \tag{1}$$

$\beta>\sqrt{2}$ is a some constant.

$Q(\mathbb{B}^2)$ is a function class $\mathbf{L}:\mathbb{B}^2\to\mathbb{R}^2_+$, which obey inequality (??) and for any $j\in\{1,2\}$ and some $R=(r_1,r_2)$, $|R|\leq\beta$:

$$\sup_{(z_1,\omega_1),(z_2,\omega_2)\in\mathbb{B}^2}\left\{\frac{l_j(z_1,\omega_1)}{l_j(z_2,\omega_2)}: |z_1-z_2|\leq\frac{r_1}{\min\{l_1(z_1,\omega_1),l_1(z_2,\omega_2)\}},\right.$$
$$\left. |\omega_1-\omega_2|\leq\frac{r_2}{\min\{l_2(z_1,\omega_1),l_2(z_2,\omega_2)\}}\right\}<\infty.$$

An analytic vector-function $F=(f_1,f_2):\mathbb{B}^2\to\mathbb{C}^2$ is said to be of bounded $\mathbf{L}$-index (in joint variables), if there exists $n_0\in\mathbb{Z}_+$ such that $\forall (z,\omega)\in\mathbb{B}^2\ \ \forall (i,j)\in\mathbb{Z}^2_+$:

$$\frac{\|F^{(i,j)}(z,\omega)\|}{i!j!l_1^i(z,\omega)l_2^j(z,\omega)}\leq\max\left\{\frac{\|F^{(k,m)}(z,\omega)\|}{k!m!l_1^k(z,\omega)l_2^m(z,\omega)}: k,m\in\mathbb{Z}_+, k+m\leq n_0\right\}.$$

The least such integer n_0 is called the $\mathbf{L}$-index in joint variables of the vector-function F and is denoted by $N(F,\mathbf{L},\mathbb{B}^2)=n_0$. Here $\|F\|=\max_{j\in\{1,2\}}\{|f_j|\}$.

The entire vector-functions having bounded index in joint variable (i.e. $\mathbf{L}(z)=(1,1)$) were considered by F. Nuray and R.F. Patterson in []. They used this function class to investigate properties of entire solutions of some systems of partial differential equations.

Theorem 1. *Let* $\mathbf{L} \in Q(\mathbb{B}^2)$. *Analytic vector-function* $F : \mathbb{B}^2 \to \mathbb{C}^2$ *has bounded* $\mathbf{L}$*-index in joint variable if and only if for every* $R \in \mathbb{R}^2$, $|R| \leq \beta$ *there exist* $n_0 \in \mathbb{Z}_+$, $p > 0$ *such that for all* $(z_0, \omega_0) \in \mathbb{B}^2$ *there exists* 2*-tuple* $(k_0, m_0) \in \mathbb{Z}_+^2$, $k_0 + m_0 \leq n_0$, *satisfying inequality*

$$\max\left\{\frac{\|F^{(k,m)}(z,\omega)\|}{k!m!l_1^k(z,\omega)l_2^m(z,\omega)} : k+m \leq n_0, (z,\omega) \in \mathbb{D}^2[(z_0,\omega_0), R/\mathbf{L}(z_0,\omega_0)]\right\} \leq$$
$$\leq p_0 \frac{\|F^{(k_0,m_0)}(z_0,\omega_0)\|}{k_0!m_0!l_1^{k_0}(z_0,\omega_0)l_2^{m_0}(z_0,\omega_0)}. \quad (2)$$

References

[1] Bandura, A., Skaskiv, O.: Analytic functions in the unit ball of bounded **L**-index in joint variables and of bounded L-index in direction: a connection between these classes, Demonstr. Math., **52** (1), 82-87 (2019). doi: 10.1515/dema-2019-0008

[2] Nuray, F., Patterson, R.F.: Vector-valued bivariate entire functions of bounded index satisfying a system of differential equations. Mat. Stud., **49** (1), 67–74 (2018). doi: 10.15330/ms.49.1.67-74

Affiliation

Vita Baksa

Department of Mechanics and Mathematics, Ivan Franko National Unviersity of Lviv, 1 Universytetska street, Lviv, Ukaine, `vitalinabaksa@gmail.com`

Andriy Bandura

Department of Advanced Mathematics, Ivano-Frankivsk National Technical University of Oil and Gas, Ivano-Frankivsk, 15 Karpatska street, Ivano-Frankivsk, Ukraine, `andriykopanytsia@gmail.com`

Oleh Skaskiv

Department of Mechanics and Mathematics, Ivan Franko National Unviersity of Lviv, 1 Universytetska street, Lviv, Ukaine, `olskask@gmail.com`