# Integrate-and-fire models with an almost periodic input function

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Integrate-and-fire (IF) neuron models are systems of the form:

$$\dot{x} = F(t,x), F : \mathbb{R}^2 \to \mathbb{R} \tag{1a}$$

$$x(t^*) = x_r \text{ if } \lim_{t \to t^{*-}} x(t) = x_T, \tag{1b}$$

where (**??**) stands for the continuous evolution of the membrane voltage between spikes (action potentials) introduced via the resetting mechanism (**??**). IF models with periodic drive (i.e. $F(t,x)$ periodic in $t$) were the subject of interest of many neuroscientists and applied mathematicians, starting from Keener, Hoppensteadt and Rinzel, who in 80's probably as the first ones noticed fruitful connection with rotation theory. The idea is hidden in the notion of the *firing map* $\Phi(t)$, whose iterates $\Phi^n(t_0)$ recover sequence of consecutive spike timings for a voltage trajectory starting at $(t_0, x_r)$. The reciprocal of its rotation number $\varrho(t_0) := \lim_{n\to\infty} \Phi^n(t_0)/n$ is equal to the *firing rate* over a trajectory $x(t; t_0, x_r)$. However, an almost-periodic stimulus definitely cannot be seen as a straightforward extension of periodic case.

The talk is based on a joint work ([J. Differential Equations 264 (2018), no.4, 2495–2537]) with Piotr Kasprzak (Adam Mickiewicz University in Poznań) and Adam Nawrocki (Poznań University of Economics and Business), where we investigate leaky integrate-and-fire (LIF) models, i.e. $F(t,x) = -\sigma x + f(t)$, driven by Stepanov and $\mu$-almost periodic functions $f(t)$. In particular, we establish a connection between IF models and the rotation theory for maps of the real line with almost periodic displacement. By allowing discontinuous inputs, we extend some previous results, showing e.g. that the firing rate for the LIF models with Stepanov almost periodic input exists and is unique. Our study also contributes to the theory of almost-periodic functions e.g. by proving that in general the mean value of a $\mu$-almost periodic function may not exist or by developing the characterization of Stepanov almost periodic functions.

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